

# Fakir Mohan Autonomous College, Balasore

UG Math, GE - I

July 8, 2021

## Questions carrying one mark each :-

1. What is the general equation of cycloid.
2. What are the asymptotes of  $xy = 1$  curve ?
3. What are the asymptotes of  $y = e^x$  ?
4. Write the equation of the sphere with centre  $(2, -1, 0)$  and radius 4.
5. Given that  $x^2 + y^2 = 4$  and  $1 \leq z \leq 2$ , identify the geometrical figure.
6. What is

$$\lim_{x \rightarrow \infty} (\sin x + \cos x) ?$$

7. What is

$$\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right) ?$$

8. What is

$$\lim_{x \rightarrow \infty} \left( 1 + \frac{1}{x} \right)^x - \lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}} ?$$

9. Is  $\tan x$  continuous for  $x \in \mathbb{R}^+$  ?
10. Is

$$f(x) = \begin{cases} 1, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

continuous ?

11. Is  $f(x) = |x|$  differentiable at  $x = 0$  ?
12. State Rolle's theorem.
13. Write the Maclaurin's series of  $\sin x$ .
14. What is the value of  $0.\infty$ ?
15. Write the Maclaurin's series of  $\log(1 + x)$ .

## Questions carrying two mark each :-

16. Find

$$\lim_{(x,y) \rightarrow (0,0)} \left( \frac{x+y}{\sqrt{x^2+y^2}} \right) ?$$

17. Find

$$\lim_{x \rightarrow \infty} (\sqrt{4x^2 + x} - 2x) ?$$

18. What is the value of the series

$$1 - \frac{9^2}{2!} + \frac{9^4}{4!} - \frac{9^6}{6!} + \dots = ?$$

19. If

$$f(x, y) = \begin{cases} \frac{x^3 + y^3}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

then  $f(x, y)$  is continuous at  $(0, 0)$  ?

20. Is  $f(x, y) = \sin(x + y)$ , then find  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  at  $(0, 0)$ .

21. Is  $f(x, y) = \sqrt{x^2 + y^4}$  homogeneous ?

22. Find the critical points of the function  $f(x, y) = x^3 + 2xy - 2x - 4y$ .

23. Maximize  $f(x, y) = x^2 + y^2$  subject to the condition  $x + y = 6 \sin(\frac{5\pi}{4})$ .

24. Find the length of  $y = \sqrt{4 - x^2}$ ,  $x \geq 0$ .

25. Find the length of the parametric curve  $x(t) = 2 \cos t$ ,  $y(t) = 4 \sin t$  for  $0 \leq t \leq \pi$ .

26. Find the area of the figure  $\frac{x^2}{4} + \frac{y^2}{16} = 1$ .

27. Solve

$$\int_{-2}^2 \int_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dy dx = ?$$

28. If  $\frac{dy}{dx} = y$ ,  $y(0) = 1$  then  $y(\ln 2) = ?$

29. Solve  $\frac{d^2 y}{dx^2} + y = 0$ .

30. Find the Wronskian of  $y_1(x) = \cos x$ ,  $y_2(x) = \sin x$ .

### Subjective Questions :

31. Trace the curve  $xy = 1$ . Find the asymptotes.

32. State and prove Lagrange's mean value theorem.

33. Find the sum of the infinite series

$$S = \frac{1}{2} - \frac{1}{3 \times 1!} + \frac{1}{4 \times 2!} - \frac{1}{5 \times 3!} + \dots ?$$

34. Check the continuity and differentiable of

$$f(x, y) = \begin{cases} \frac{x^2 + y^2}{\sqrt{x^4 + y^4}}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

35. Max :  $z = x^2 + y^2$ , subject to the condition  $x - 2y = 4$ .

36. Calculate

$$\int_0^1 \int_{2y}^2 e^{x^2} dx dy.$$

37. Solve  $\frac{d^2 y}{dx^2} + y = 0$ ,  $y(0) = y(\pi)$ ,  $y' = y(x)$ .

..... Good Luck .....