Fakir Mohan Autonomous College, Balasore

UG Math, CC - VI
Group Theory - I

July 20, 2021

Questions carrying one mark each :-

1. Write the basic structure 0f dihedral group D_4 of order 8.

2. Show the figure of D_4 and D_3 of order 8 and 6 respectively?

3. Write the elements of quaternion group Q_4.

4. Define permutation group S_n. Construct S_2.

5. Define group. Is $(\mathbb{Z}, .)$ is group under multiplication ?

6. Show that $(\mathbb{Z}_3, +)$ is a group.

7. What is the difference between $Gl_n(\mathbb{R})$ and $Sl_n(\mathbb{R})$?

8. Is set of integers is a group under substraction.

9. Is $\mathbb{Z}_4$ is a group under multiplication modulo 4 ?

10. Define order of a group. What is the order of S_4 , where S_4 is the permutation group of 4 symbols.

11. What is inverse of $a = (1\ 2\ 3)(4\ 5)$? here $a \in S_5$, where S_5 is the permutation group of symbols.

12. Define order of an element in a group. What is the order of Q, where Q is set of rational numbers.

13. Number of elements of order 4 in $(\mathbb{Z}_3, +)$?

14. What is the order of 3 in $(\mathbb{Z}_4, +)$?

15. Define abelian group, give an example of a non cyclic abelian group.

Questions carrying two mark each :-

16. Give an example of a non abelian cyclic group.

17. (True/ False) If a commutes with b, then a^{-1} is also commutes with b^{-1}?

18. Define generators of a cyclic group.

19. Define centralizer of a group.

20. Define normalizer of a group.

21. Define coset of group?

22. $f = (1\ 3\ 5\ 7\ 9)$ is an even permutation or odd permutation?

23. Define a normal subgroup.

24. Find the order of the permutation $f = (1\ 2)(3\ 4\ 5)$.

25. What is the factor group?

26. Show that $f : \mathbb{Z} \to \mathbb{Z}$ define by $f(x) = kx$ (k $\in \mathbb{Z}$) is homomorphism.

27. $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = 2x$ is homomorphism.

28. What is the isomorphism in a group ?

29. What is the automorphism in a group ?

   Subjective Questions :

30. If $G = (\mathbb{Z}_2, +) \times (\mathbb{Z}_4, +)$. Find possible order of elements in G.

31. Let G be a group and $a \in G$, then $o(a) = o(xax^{-1}) = o(x^{-1}ax)$, where $x \in G$.

32. Let $a \in G$ and $a^n = e$ thenprive $o(a)|n$.

33. Show that $o(a) = o(a^{-1})$, where $a \in G$.

34. If G be a finite group and H is a subgroup of G then $o(H)|o(G)$.

35. Find the numbers of elements of order 2 and 5 in S_5.

36. State and prove Fundamental theorem of homomorphism.

37. If G is abelian group and H is subgroup of G, then every left coset of H in G is right coset of H in G.

.... Good Luck ....