Fakir Mohan Autonomous College, Balasore

UG Math, CC - I
Calculus - Paper - I

1. Write $\sinh(x) + \cosh(x)$ in exponential form.

2. Is the function $y = |x|$ differentiable ?

3. Is $y = \sqrt{x}$ differentiable on $x \in \mathbf{R}^+$?

4. Is $y = \sqrt{x^2}$ differentiable ?

5. If
$$f(x) = \begin{cases} x^2 \sin(\frac{1}{x}), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

   Then $f'(x)$ is continuous at $x = 0$. (True/ False).

6. Find the
$$\lim_{x \to 0}(1 - \frac{1}{1+x})^x \ ?$$

7. Find the sum of the series
$$\frac{1}{3} + \frac{2}{4} + \frac{1}{27} + \frac{2}{81} + \frac{1}{243} + \frac{2}{724} + ...?$$

8.
$$\lim_{x \to \infty}(\sqrt{4x^2 + x} - 2x) = ?$$

9.
$$\lim_{x \to \infty} -5\cos(x) = ?$$

10. Find the concavity of the function $\sin(x)$ for $x \in [-\pi, \pi]$, concave up for $x \in$ ....?

11. Find
$$\int_0^{\pi} \sin^3(\frac{x}{2})dx = ...?$$

12. Find
$$\int_0^{\frac{\pi}{2}} \sin^7(x)\cos^5(x)dx = ...?$$

13. Find the distance travelled by the particle on the parametric curve $x(t) = t, y(t) = t^2$ for $1 \leq t \leq \sqrt{2}$.
    Answer : ......?

14. Find the area of the region that is enclosed between the curve $x^2 + y^2 = 4$ above and the x-axis. ....?

15. Find the volume of the sphere with centre $(1, 0, 0)$ and radius 2.

16. What is the projection of the vector $< 1, -2, 0 >$ on the vector $< 1, 0, 0 >$.

17. What is the angle between two vectors $< 1, -2, 0 >$ and $< 0, 0, 1 >$.

18. What is $\vec{a} \times (\vec{b} \times \vec{c})$, if $\vec{a}$ and $\vec{c}$ are parallel.

19. Angle between the vector $< 1, 2 >$ and $< 1, 0 >$ is ....

20. Find the polar equation of the curve with centre $(2, 0)$ and radius 4.

21. $\int_{-2}^{2} |x| dx = ?$

22. $\int_{1}^{2e} \ln(x) dx = ?$

23. Find the assymptotes of the function $y = \frac{1}{x}$.

24. Find the arc length of $\vec{r}(t) = < b + t^2, 2t, 0 >, 1 \leq t \leq 4$.

25. If $\vec{r}(t) = < \frac{t^2}{2}, 2 - 5t, 9 >$, Find the tangent vector $\vec{T}$.

26.
$$\lim_{x \to 0} \frac{1 - 2x + 2x^2 - e^{-2x}}{x^3} = ?.$$

27. If $f(x) = \sin^{-1}(x)$, find $f''(1)$.

28. Find the inflection points of $\cos(x)$ for $x \in [-\pi, \pi]$.

29. Find the right Riemann sum for $f(x) = x^2$, for $x \in [1, 4]$ with step size 1.

30. Find the equation of the parabola with vertex (1,-1) and passes through origin.

31. Find the area of the surface revolution of $y^2 = 2x$ for $1 \leq x \leq 4$.

32. Find the equation of the ellipse $\frac{x^2}{4} + \frac{y^2}{16}$ in polar form.

33. Find all vector V such that $< 1, 2, 1 > \times V = < 3, 1, -5 >$.

34. If $a.b = \sqrt{3}$ and $a \times b = < 1, 2, 2 >$, find the angle between a and b.

35. Find the volume of parallelopiped determine by the vectors a, b and c, where $a = < 1, 2, 3 >, b = < -1, 1, 2 >, c = < 2, 1, 4 >$.

Subjective Questions :

1. Find the assymptotes of the parametric curve $x(t) = \frac{t^2}{t^3+1}$ and $y(t) = \frac{t^2+2}{t+1}$ and plot the graph of $\cosh x$.

2. Find the value of the below definite integral
$$\int_{0}^{\infty} e^{-\frac{x^2}{2}} dx.$$

3. Find the area enclosed by the region, $y = e^x, y = e^{2x}, x = 0, x = \ln(2)$.

4. Find the volume of the solid generated by the region between the graphs of the equations $f(x) = \frac{1}{2} + x^2$ and $g(x) = x$ over the interval $[0, 2]$ is revolved about the x-axis.

5. Find $\int_0^1 \int_x^1 \sin(y^2) dy dx = ?$

6. $(\int_0^1 x^4(1 - x)^5 dx)^{-1} = ?$

7. If $y(x) = \int_{\sqrt{x}}^{x} \frac{e^t}{t} dt, x > 0$, then $y'(1) = ?$

8. Show that the volume of the cylinder $x^2 + y^2 \leq 4, 0 \leq z \leq 3$ is 12π.

9. For $x > 1, f(x) = \int_1^x (\sqrt{\log t} - \frac{1}{2} \log \sqrt{t}) dt$. The number of tangent to the curve, $y = f(x)$ parallel to the line $x + y = 0$ is ....

10. Show that $y = \frac{1}{x}$ is an equation of hyperbola by converting it into it's standard form.

.... Good Luck ....